Tests of a Method for Making Geodetic Ties by Observing a Satellite Optical Beacon

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The intervisible technique uses simultaneous optical observations of rockets or satellites for relative positioning of reference and unknown observers. The technique is developed using the method of maximum likelihood. Ten Air Force PC-1000 cameras in the southeastern United States participated in a program of simultaneous photographic observations of flashes from the ANNA 1-B geodetic satellite. Geodetic position determinations using these ANNA observations indicate that the PC-1000 camera system used with the intervisible technique is capable of extending geodetic control with an accuracy of about 10 m.

Introduction

THE ANNA 1-B geodetic satellite was launched on October 31, 1962. The satellite carries Army Secor range and Navy Doppler range rate equipment and the Air Force contribution, a flashing light system. Many photographic observations have been made of the ANNA flashes, and a number of different techniques are applicable for reducing these data into geodetically significant results. The reduction techniques either rely on dynamic theory or are purely geometric. In the latter class are long-line azimuth determinations and the basis of this paper, the intervisible technique.

The term "intervisible" implies that directional measurements of common flashes are made simultaneously from several stations on the earth. The object of the technique is to solve for the coordinates of unknown stations or to refine the coordinates of known stations and, if applicable, to provide interdatum translations.

From September 1963 until January 1964, 10 PC-1000 camera stations in the southeastern United States participated in a series of ANNA optical observations called the Gulf Test. The purpose of the Gulf Test is to prove out a a stereo-triangulation system that encompasses a satellite-borne flashing light source, PC-1000 camera systems, a plate reduction method developed specifically for long focal length cameras,[1] and a computer program for intervisible geodetic reductions.[2]

In the Gulf Test, as in the whole ANNA project, the Air Force has utilized the optical portion of the ANNA geodetic satellite program through the participation of three agencies: Air Force Cambridge Research Laboratories (AFCRL), Air Photographic and Charting Service (APCS), and the Aeronautical Chart and Information Center (ACIC). AFCRL computed the look-angle information, scheduled the flash sequences, triggered the flashes with an emergency override system when necessary, and performed the final geodetic reductions. APCS distributed the look-angles and operated and maintained the PC-1000 stellar cameras. ACIC performed the plate reductions, forwarding right ascensions and declinations of the flash positions to AFCRL for use in the geodetic reductions.

Satellite Beacon

The flashing beacon in the ANNA satellite was developed by Edgerton, Germeshausen, and Grier Inc. It consists of two pairs of xenon-filled lamps mounted in reflectors, one pair on the north face of the solar cell panel and one pair on the south face. The satellite is magnetically oriented so that one pair of lights serves as the beacon when the satellite is north of the magnetic equator and the other serves as the beacon when it is south of the magnetic equator (see Fig. 1).

A flash sequence is composed of five flashes at 5.6-sec intervals. The energy per flash is 1100 joules. The flash duration is 1.2 msec, and the light output is about 8800 candle-sec/flash. Under normal operation, flash times are inserted into the satellite memory from an injection station located at the Applied Physics Laboratory in Maryland when the satellite passes within range of the station. The timing of the flash sequences is controlled by the memory clock. At the predetermined times, the clock initiates the flash sequences to an accuracy of 0.1 to 0.5 msec.

An alternate method of initiating flash sequences is afforded by the emergency override system (EMOS), which was located at AFCRL, Bedford, Mass., during the Gulf Test exercise. The EMOS enables an operator to initiate flash sequences on command to 1-msec accuracy. The unreliability of ANNA clock timing necessitated using the EMOS for executing flash sequences after October 11, 1963.

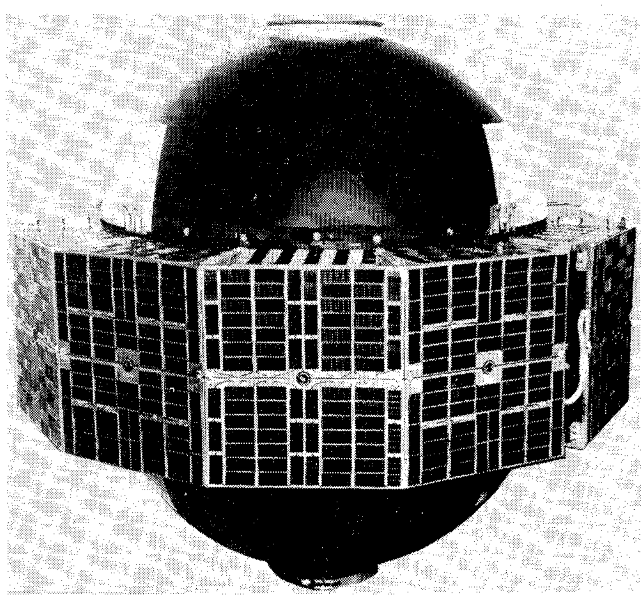

Fig. 1 The ANNA 1-B geodetic satellite. On the equatorial band of solar cells can be seen two flash-tube assemblies aimed at the satellite's magnetically oriented north pole. Identical units aim at the satellite's south pole.

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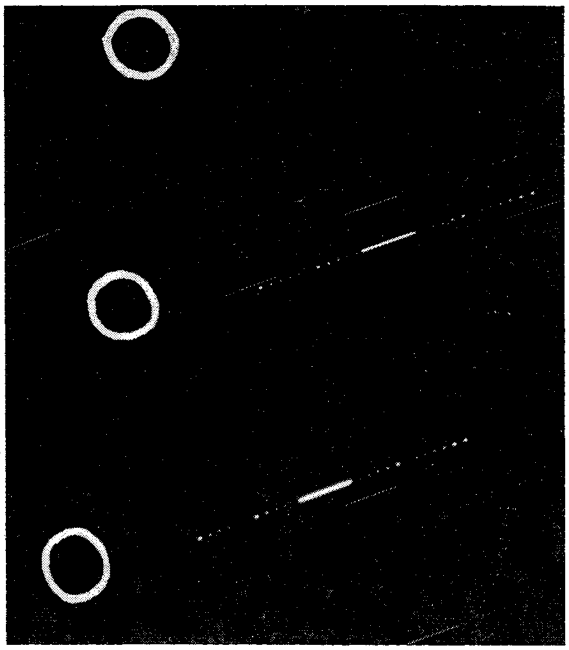

Fig. 2 On this 5 × 5 cm positive print of a portion of a PC-1000 plate, the ANNA flashes are circled and are barely discernible. The star trails are shutter-chopped during the calibrations that precede and follow the open period during which the satellite is programed to flash.

PC-1000 Camera System

The PC-1000, a fixed stellar camera, has a 1000-mm focal length, a 200-mm aperture, and a 10° square field of view. Kodak 103F emulsion photographic plates were used to photograph the ANNA flashes simultaneously against star backgrounds that included stellar magnitudes as low as the ninth.

A normal observation is made as follows. Two precalibration sequences of shutter openings separated by approximately 40 sec are taken with openings of 2, 1, 0.5, 0.3, and 0.1 sec, the shutter being closed for 15 sec between openings. Each recorded star gives rise to a trace of point-like images. Fifteen seconds after the precalibration, the shutter is opened for 1 min to photograph the ANNA five-flash sequence, which produces flash images ranging from 40 to 70 μ in diameter, depending on the camera-satellite range. After the shutter is closed for a 15-sec interval, a series of double postcalibrations is taken. Figure 2 is a positive enlargement of a 2-in.-square area of a PC-1000 plate with the ANNA flash images circled.

The final plate consists of a series of five flash images recorded on a background of hundreds of control points provided by the star images. The control points selected are those that closely match the photographic characteristics of the flash images. This eliminates the measuring bias problem, since the bias is the same for both the flash images and the measured control. The star background allows each camera to be precisely calibrated for its elements and for its orientation in space. Because the rays from the flashes to the cameras are refracted the same amount as the rays from the stars to the cameras, the effects of refraction are removed by only minor parallactic corrections. An ANNA plate reduction usually involved 60 to 70 stellar images corresponding to the double precalibration and postcalibration exposure times.

The flash directions from each observing station are determined from the plate measurements of control points and flash images. These are forwarded by ACIC to AFCRL in the form of topocentric right ascensions and declinations. Included with the flash directions are their standard deviations, which are by-products of the plate reduction program. These, together with the observing station positions and their assumed uncertainties, comprise the input for the intervisible geodetic reduction program.

Intervisible Adjustment Theory

The usual departure point for the adjustment of overdetermined systems, such as those stemming from the Gulf Test, is the method of least squares. This method is based rigorously on the method of maximum likelihood for normal deviates, which is the foundation on which the reduction technique for the Gulf Test systems is directly constructed.

The principle of the method of maximum likelihood is to maximize the likelihood function or its logarithm L with respect to the parameters to be adjusted, ϑ^p, $p = 1, 2, \ldots, P$. Consider these parameters as the elements of a P-vector θ. Then the conditions for a stationary point at $\theta = \hat{\theta}$ can be written as

$$(\partial L/\partial\vartheta^p)_{\hat{\theta}} = 0 \qquad p = 1, 2, \ldots, P$$

Suppose that $\tilde{\theta}$ is an approximation to θ. By Taylor series expansion to the first order in $(\hat{\vartheta}^p - \tilde{\vartheta}^p)$, these conditions become

$$\left(\frac{\partial L}{\partial\vartheta^p}\right)_{\tilde{\theta}} + \sum_{q=1}^{P}\left(\frac{\partial^2 L}{\partial\vartheta^p\partial\vartheta^q}\right)_{\tilde{\theta}}(\hat{\theta}^q - \tilde{\vartheta}^q) = 0$$

$$p = 1, 2, \ldots, P$$

Let the inverse of the matrix $[-(\partial^2 L/\partial^p\vartheta\partial\vartheta^q)_{\tilde{\theta}}]$ be $[\eta^{pq}]$.

The first-order solution for $\hat{\theta}$ becomes

$$\vartheta^p = \vartheta^p + \sum_{q=1}^{P}\eta^{pq}\left(\frac{\partial L}{\partial\vartheta^q}\right)_{\tilde{\theta}} \qquad p = 1, 2, \ldots, P \quad (1)$$

This array is the form used in the iterative solutions of the Gulf Test networks. It can be shown that, with such an iterative technique, $\hat{\theta}$ converges in probability to the true parameter θ, and η^{pq} converges in probability to the expected value of $(\hat{\vartheta}^p - \vartheta^p)(\hat{\vartheta}^q - \vartheta^q)$, that is, $[\eta^{pq}]$ is the solution covariance matrix.

The elements of the parameter vector θ for the intervisible networks of the Gulf Test are taken as the coordinates of R flash positions x_i^r, $r = 1, 2, \ldots, R$, and the coordinates of S ground stations, y_i^s, $s = 1, 2, \ldots, S$. (Subscripts always pertain to the three-vector of cartesian space, and the convention of summation over repeated subscripts is adopted.) Estimates are available before the reduction of the coordinates of the stations $y_i^{0s} \approx y_i^s$ and of their uncertainties. The uncertainties for each station are normally distributed with zero means and inverse covariance elements K_{ij}^s.

It is assumed that there is no correlation between stations and that each error ellipsoid defined by K_{ij}^s is an ellipsoid of revolution about the local vertical axis. Properly interpreted, the solution to be derived is valid in the limit as any error ellipsoid becomes infinitesimal (fixed station) or infinite (completely unknown station).

The range from station s to flash r is

$$w^{rs} = |x_i^r - y_i^s|$$

and the direction cosines from the station to the flash are

$$\mu_1^{rs} = (x_i^r - y_i^s)/w^{rs}$$

The corresponding observed direction cosines are ν_i^{rs}. It is assumed that the angle φ^{rs} between the vectors $[\mu_i^{rs}]$ and $[\nu_i^{rs}]$ is the resultant of two angles that are mutually orthogonal on the celestial sphere, and each of which is normally distributed with zero mean and standard deviation σ^{rs}. For a missing observation, $1/\sigma^{rs}$ is taken as zero. Because φ^r

is small ($\sim 10^{-5}$),

$$(\varphi^{rs})^2 = \sin^2\varphi^{rs} = 1 - \cos^2\varphi^{rs} = 1 - (\mu_i^{rs}\nu_i^{rs})^2$$

The log likelihood function for this model is

$$L = \text{const} + \frac{1}{2}\sum_{r=1}^{R}\sum_{s=1}^{S}\left(\frac{\mu_i^{rs}\nu_i^{rs}}{\sigma^{rs}}\right)^2 - \frac{1}{2}\sum_{s=1}^{S}(y_i^s - y_i^{0s})K_{ij}^s(y_j^s - y_j^{0s})$$

Its first derivatives are, to the first order in φ^{rs},

$$\frac{\partial L}{\partial x_i^r} = \sum_{s=1}^{S}\frac{\nu_i^{rs} - \mu_i^{rs}}{w^{rs}(\sigma^{rs})^2}$$

$$\frac{\partial L}{\partial y_i^s} = \sum_{r=1}^{R}\frac{\nu_i^{rs} - \mu^{rs}}{w^{rs}(\sigma^{rs})^2} - K_{ij}^s(y_i^s - y_j^{0s})$$

Its second derivatives are, to the zeroth order in $\varphi^{rs}(\mu_i^{rs} = \nu_i^{rs})$,

$$\partial^2 L/\partial x_i^r \partial y_j^s = (\delta_{ij} - \mu_i^{rs}\mu_j^{rs})/(w^{rs}\sigma^{rs})^2$$

$$\frac{\partial^2 L}{\partial x_i^r \partial x_j^r} = -\sum_{s=1}^{S}\frac{\delta_{ij} - \mu_i^{rs}\mu_j^{rs}}{(w^{rs}\sigma^{rs})^2}$$

$$\frac{\partial^2 L}{\partial y_i{}_s \partial y_j^s} = -\sum_{r=1}^{R}\frac{\delta_{ij} - \mu_i^{rs}\mu_j^{rs}}{(w^{rs}\sigma^{rs})^2} - K_{ij}^s$$

where δ_{ij} is the Kronecker delta. All other second derivatives are zero. The second derivatives are calculated from the first approximations for x_i^r and y_i^s, and the covariance matrix is found by inversion. For reasonable first approximations, this matrix need be evaluated only once. The first derivatives are multiplied by the covariance matrix as in Eq. (1) to generate corrections to x_i^r and y_i^s (that is, to ϑ^p), which are in turn used to update the first derivatives for iterative application of the equation. Iterations are continued until all of the corrections generated are significantly less than the solution variances. The criterion that

$$(\hat{\vartheta}^p - \tilde{\vartheta}^p) < 100\ \eta^{pp} \qquad p = 1, 2, \ldots, P$$

is usually satisfied by the second iteration of the Gulf Test solution; one application of the first-order solution is therefore usually quite sufficient. If station s is completely unknown, $K_{ij}^s = 0$, and the preceding formulas need no modification. If, however, the station is fixed, set $K_{ij}^s = \alpha\delta_{ij}$ and let $\alpha \to \infty$. Then $\partial^2 L/\partial y_i^s \partial y_j^s \to -\alpha\delta_{ij}$, and the corresponding elements in the covariance matrix approach $\alpha^{-1}\delta_{ij}$. These elements multiply only $\partial L/\partial y_i^s \to -\alpha(y_i^s - y_i^{0s})$ in the application of Eq. (1); the net result is that any y_i^s are replaced by y_i^{0s} after one iteration, and they are not modified thereafter. In practice, this is accomplished simply by dropping y_i^s as adjustment parameters. The foregoing solution is valid only for intradatum ties, but it can easily be extended for interdatum ties as well. For instance, suppose that, for $s = 1, 2, \ldots, T$, y_i^{0s} is referred to one datum, and for $s = T+1, T+2, \ldots, S$, y_i^{0s} is referred to a second datum. Let the datum displacement be z_i, and in all of the preceding formulas replace y_i^{0s} by $y_i^{0s} + z_i$, when $s > T$.

The additional derivatives with respect to z_i of the modified log likelihood function are, to the appropriate order,

$$\frac{\partial L}{\partial z_i} = \sum_{s=T+1}^{S} K_{ij}^s(y_j^s - y_j^{0s} - z_j)$$

$$\frac{\partial^2 L}{\partial z_i \partial z_j} = -\sum_{s=T+1}^{S} K_{ij}^s$$

$$\frac{\partial^2 L}{\partial z_i \partial y_j^s} = K_{ij}^s \qquad s > T$$

All other additional derivatives are zero. With the modified and new derivatives, the datum displacement can be carried along with the coordinates of the stations and flashes in the parameter vector θ.

An advantage of this type of solution over the usual least squares approach is that, if the geometry and precision of a network and its measurements are specified, the covariance matrix can be calculated without requiring actual observations. Thus, error analyses can be performed for hypthetical models. For the Gulf Test measurements, the observed apparent right ascensions and declinations and their standard deviations are calculated in the least squares plate reductions of ACIC. These standard deviations are checked against the solution errors (that is, the magnitudes of the cross products of the solution and observed vectors); rarely do they require modification and second solutions. In such cases, there are slight differences between the solution parameters, but the covariance matrices change moderately.

Observation Network

The 11 PC-1000 camera sites occupied during the Gulf Test are listed in Table 1. Station 650, Jacksonville, and Station 650M, Mulberry, were occupied by the same camera, the camera being at Jacksonville for the first half of the test program. The 1381st Geodetic Survey Squadron of APCS provided the coordinates of the camera sites, which were tied to first-order control of the North American Datum 1927 (NAD 27); ACIC supplied the geoid-spheroid separations necessary to convert heights above mean sea level to heights above the reference spheroid.

Thirty-nine observation groups were obtained in the Gulf Test. (A group is defined here as a flash sequence successfully photographed from three or more camera stations.)

Table 1 Geodetic coordinates of camera sites, NAD 27

Geographic site	Station no.	Latitude north, ϕ	Longitude west, λ	Height above mean sea level, h, m	Height above spheroid, H, m
Houma, La.	640	29° 33′ 44″.80	90° 40′ 44″.19	2.0	7.0
Ellington, Tex.	641	29° 35′ 39″.89	95° 09′ 14″.04	8.2	12.2
England, La.	643	31° 19′ 15″.91	92° 31′ 31″.91	26.8	29.8
Corpus Christi, Tex.	644	27° 41′ 20″.88	98° 14′ 36″.88	3.4	6.4
Dauphin I, Ala.	647	30° 14′ 48″.28	88° 04′ 42″.51	1.2	5.2
Hunter, Ga.	648	32° 00′ 05″.87	81° 09′ 13″.64	12.2	12.2
Jupiter, Fla.	649	26° 57′ 12″.57	80° 04′ 55″.80	6.8	9.3
Jacksonville, Fla.	650	30° 14′ 10″.72	81° 40′ 57″.46	5.8	9.3
Mulberry, Fla.	650M	30° 13′ 05″.46	81° 41′ 47″.81	6.4	9.9
Orlando, Fla.	686	28° 34′ 26″.03	81° 19′ 39″.07	28.6	33.7
Orlando, Fla.	686C	28° 34′ 26″.70	81° 19′ 38″.65	28.6	33.7

Table 2 Station observations (ANNA satellite altitude approximately 1100 km)

Group no.	Date	Time GMT	Sub-sat. lat., °N	Sub-sat. lat., °W	Flash mode	640	641	643	647	648	649	650M
8	Oct. 1, 63	0955	33	85	Memory		x	x	x	x		
19	Oct. 22, 63	0347	31	87	EMOS	x		x	x	x		
22	Oct. 26, 63	0306	26	88	EMOS		x	x	x	x		
28	Dec. 19, 63	0228	33	90	EMOS	x		x	x	x	x	x
30	Dec. 20, 63	0149	31	88	EMOS	x			x	x	x	x
34	Jan. 3, 64	0601	28	92	EMOS	x	x	x	x	x	x	x

Numerical results of this paper are based on available data (Table 2) that were obtained from six groups and seven cameras. They are plotted in Fig. 3.

Results

Five geodetic networks were examined with the intervisible computer program. In a network, each camera site was considered either as a "reference" station or as a "free" station. If the coordinates of a reference station were not parameters of the adjustment, the station was considered "fixed", if the coordinates of a reference station were constrained parameters of the adjustment, the station was considered "adjustable."

The horizontal standard deviations of the free stations were uniformly chosen as 100 m. Except for one case in which the initial coordinates of a free station were deliberately in error by 140 m, these constraints were relatively so weak that they were tantamount to assuming infinite horizontal standard deviations. The horizontal constraints were rather harsh for the station deliberately in error, but their effects were offset by the strength of the observations, coupled with the assumption that the two reference stations for that case were fixed. The vertical standard deviations of the free stations were uniformly chosen as 5 m. These are reasonable uncertainties for the geoid-spheroid separations in the Gulf Test area. Because of the poor vertical control inherent in most of the Gulf Test networks, the vertical standard deviations were usually decreased only slightly in the adjustments, that is, tight vertical constraints were usually necessary to keep the vertical coordinates from "running away."

Network 1

The coordinates of the free station, 640, were determined from intervisible observations of 10 flashes in groups 30 and 34, using the fixed reference stations 648 and 649 as a baseline. The observation standard deviations were 0.9 for station 648, 1″1 for station 649, and 1″6 for station 640. The free station was intentionally given an initial position error of +3″ in latitude and −4″ in longitude.

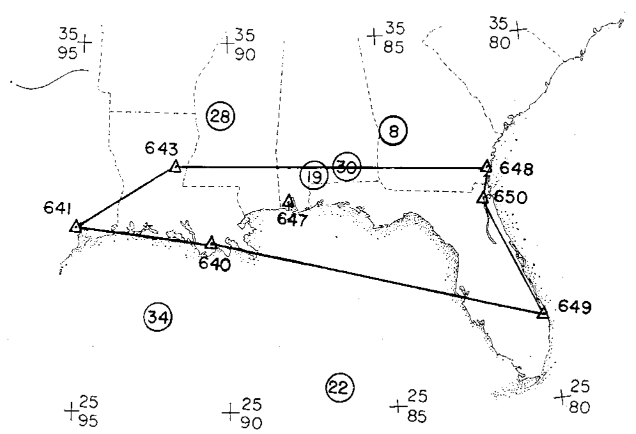

Fig. 3 Seven camera sites, represented by triangles, observed six groups of ANNA flashes, represented by circles, to provide data for the computer runs described in this paper.

The sizeable free station position error was eliminated in the adjustment that is summarized in Table 3. The distance ΔR between the NAD 27 position and the ANNA determined position is 7.1 m. The square root of the trace of the solution covariance matrix for the free station is $\sigma_R = 8.7$ m.

Network 2

The coordinates of the free station, 648, were determined from intervisible observations of nine flashes in groups 8 and 22, using the fixed reference stations 641 and 643 as a baseline. The observation standard deviations were 0″7 for station 643 and 0″8 for stations 641 and 648.

Table 3 Geodetic position determination of station 640 from stations 648 and 649[a]

	φ	λ	H m	R m
North American datum 27	29° 33′ 44″80	90° 40′ 44″19	7.0	...
ANNA data reduction	44″78	43″93	7.6	...
Δ	0″02	0″26	−0.6	7.1
σ	0″11	0″25	4.8	8.7

[a] Fixed reference stations, 648 and 649; free station, 640. Input error station 640; $\varphi = +3''$, $\lambda = -4''$. Observations: all stations observed five flashes each on groups 30 and 34. $\Delta R = [(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]^{1/2}$; $\sigma_R = [\sigma_x{}^2 + \sigma_y{}^2 + \sigma_z{}^2]^{1/2}$.

The network geometry of this case is not favorable for a strong determination of the horizontal coordinates of the free station. The discrepancy ΔR is 13.4 m ($\sigma_R = 12.0$ m). The vertical standard deviation of the free station decreased from 5.0 to 3.8 m. Compared with other runs, this indicates a relatively favorable geometry for vertical control. If there had been no vertical constraint, the vertical standard deviation of the solution would have been approximately 6 m.

Of incidental interest are the uncertainties of the final flash positions. The flash error ellipsoids of this reduction are

Table 4 Geodetic position determination of station 648 from stations 640 and 643[a]

				H, m	R, m,
North American datum	27	32° 00′ 05″87	81° 09′ 13″64	12.2	...
ANNA data reduction	1	05″76	14″74	10.4	...
	Δ	+0″11	−0″83	+1.8	21.7
	σ	0″07	0″50	4.9	14.6
ANNA data reduction	2	05″76	14″44	10.5	...
	Δ	+0″11	−0″80	+1.7	21.3
	σ	0″11	0″83	4.9	22.2
ANNA data reduction	3	05″80	14″26	10.5	...
	Δ	+0″07	−0″62	+1.7	16.5
	σ	0″22	1″30	4.9	35.1

[a] Reference stations, 640 and 643; free station, 648. Run 1: fixed. Run 2: $\sigma_{N\text{-}S} = \sigma_{E\text{-}W} = 3$ m, $\sigma_H = 0.1$ m. Run 3: $\sigma_{N\text{-}S} = \sigma_{E\text{-}W} = 6$ m, σ_H 0.1 m. Observations: all stations observed four flashes on group 19 and three flashes on group 28. $\Delta R = [(\Delta x)^2 + (\Delta y)^3 + (\Delta z)^2]^{1/2}$; $\sigma_R = [\sigma_x{}^2 + \sigma_y{}^2 + \sigma_z{}^2]^{1/2}$.

Table 5 Geodetic position determination of stations 640, 641, 643, and 647 from stations 648, 649, and 650[a]

	Reduction				North American datum ANNA data reduction								
Sta.	σ_φ	σ_λ	σ_H, m	σ_R, m	$\Delta\varphi$	$\Delta\lambda$	ΔH, m	Δx, m	Δy, m	Δz, m	R, m	Distance, km	Proportional accuracy, NAD standard
640	0".11	0".21	4.7	8.3	+0".09	+0".29	−0.2	−7.7	+1.6	+2.0	8.1	987	1/122,000
641	0".14	0".36	5.0	11.7	−0".06	+0".61	0.0	−16.8	+0.7	−1.4	16.9	1454	1/86,000
643	0".11	0".29	5.0	9.7	+0".09	+0".33	−0.4	−8.3	+2.5	+2.5	9.0	1238	1/138,000
647	0".07	0".18	4.6	6.9	+0".14	+0".42	+0.7	−11.3	+1.5	+4.5	12.3	761	1/62,000

[a] Fixed reference stations, 648, 649, and 650; free stations 640, 641, 643, and 647. Observations: stations 648, 649, 650, 640, and 647 observed five flashes on group 30 and five flashes on group 34. Stations 641 and 643 observed five flashes on group 34. Stations 641 and 643 observed five flashes on group 34. $\Delta R = [(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]^{1/2}$; $\sigma_R = [\sigma_x^2 + \sigma_y^2 + \sigma_z^2]^{1/2}$.

cucumber-shaped, and the longest axis of each is aligned in the general direction of the reference baseline from the flash locations. For flashes in group 8, the standard deviations along the longest axes and normal to these axes are about 15 and 2.5 m. For flashes in group 22, these standard deviations are about 13 and 3.0 m.

Network 3

The coordinates of the free station, 648, were determined from intervisible observations of seven flashes in groups 19 and 28, using the reference stations 640 and 643 as a baseline. The observation standard deviations were 1".3 for station 640 and 0".8 for stations 643 and 648.

Three adjustments, which were made with the data, are summarized in Table 4. For the first run, the reference stations were considered fixed, and for the second and third runs they were adjustable. The reference stations for the second run were given standard deviations of 3 m horizontally and 0.1 m vertically, which still overstate the accuracies of NAD 27 first-order stations. For the third run, the horizontal standard deviations were doubled to 6 m, and the vertical standard deviations were unchanged.

For the first run, the discrepancy at the free station is $\Delta R = 21.7$ m ($\sigma_R = 14.6$ m); for the second run, the discrepancy is about the same, $\Delta_R = 22.2$ m ($\sigma_R = 22.2$ m); for the third run, the discrepancy is $\Delta R = 16.5$ m ($\sigma_R = 35.1$ m). In each run, the major portion of the free station error is in its longitude; this is understandable because of the long, narrow triangle formed by the stations involved. The adjustable reference stations have discrepancies $\Delta R = 1$ m for the second run and $\Delta R = 2$ m for the third run; the covariances of the adjustable reference stations were not appreciably decreased by the adjustments.

Network 4

The coordinates of the four free stations (640, 641, 643, and 647) were determined from intervisible observations of 10 flashes in groups 30 and 34, using the three fixed reference stations (648, 649, and 650). Ten flashes were observed by all stations except 641 and 643, which observed only the five flashes of group 34. The observation standard deviations were 0".8 for station 643, 0".9 for station 648, 1".6 for station 640, 1".1 for stations 649 and 650, and 1".2 for stations 641 and 647.

The adjustment is summarized in Table 5. The free station discrepancies range between 8.1 and 16.9 m.

Table 6 Geodetic position determination of station 647 from stations 640, 641, 643, 648, 649, and 650M[a]

		φ	λ	H, m	R, m
North American datum	27	30° 14′ 48".28	88° 04′ 42".51	5.2	...
ANNA data reduction		48".16	42".38	4.8	...
	Δ	0".12	0".13	0.4	5.2
	σ	0".03	0".03	2.8	3.2

[a] Fixed reference stations, 640, 641, 643, 649, and 650 M; free station, 647. Observations: 1) all stations observed five flashes on group 34; 2) stations 641, 643, and 647 observed three flashes on group 8 and five flashes on group 22; 3) stations 640, 643, 648, and 647 observed four flashes on group 19; 4) stations 640, 643, 648, and 647 observed two flashes on group 28; 5) stations 640, 648, 649, 650M, and 647 observed five flashes on group 30. $\Delta R = [(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]^{1/2}$; $\sigma_R = [\sigma_x^2 + \sigma_y^2 + \sigma_z^{1/2}]$.

Network 5

In the final adjustment, the coordinates of the free station, 647, were determined from intervisible observations of 24 flashes in groups 8, 19, 22, 28, 30, and 34, using the six surrounding fixed reference stations (640, 641, 643, 648, 649, and 650M). The observation standard deviations were 0".9 for station 647, 1".1 for station 650M, 2".1 for station 640, 0".8 for stations 643 and 648, and 1".0 for stations 641 and 649.

The geometry is favorable for a strong determination of the coordinates of the free station. The adjustment is summarized in Table 6. The free station discrepancy is $\Delta R = 5.2$ m ($\sigma_R = 3.2$ m).

After this adjustment, the sample standard deviation for all observation residuals is 1".2. Statistical chi-square and variance ratio tests indicate that the data are consistent with the original assumption that the observation angular errors are normally distributed, and the data are consistent with the hypothesis that the observation standard deviation is independent of the station and flash.

Conclusions

ANNA 1-B, the first satellite specifically instrumented for geodetic purposes, was part of a research program that has demonstrated the feasibility of using photographic satellite surveying techniques to derive significant geodetic data. One phase of this program was the Gulf Test in which measurements of photographs from PC-1000 camera systems were processed eventually to yield camera station coordinates. The discrepancies between the Gulf Test coordinates and those of the NAD 27 are typically of the order of 10 m.

Parallel tests, encompassing different photographic systems and data reduction techniques, have been successfully performed by the Smithsonian Astrophysical Observatory and NASA. Many observations of the Navy Doppler System have been collected[3] which have also yielded geodetically significant results. Although the Army Secor System was inoperative on the ANNA satellite, in subsequent geodetic satellites Secor has demonstrated its value as a geodetically useful system.

References

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